Estimation of the concentration field generated by a moving gaseous source via mobile sensors: Towards the convergence of Control Theory and CFD

WPI
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summary

- Develop computational framework for the estimation of the state (concentration profile) and proximity of the gaseous source.

- Implement a method that provides a real-time solution (process-state estimate) of the 3D atmospheric advection diffusion process.

- Provide the guidance of a sensing aerial vehicle for the effective estimation of the process state and moving source proximity.
Research approach

• Develop a model-based estimation scheme that provides the sensor’s spatial relocation employed to find the proximity of the moving source.
  – sensor spatial relocation is accomplished by means of an SAV that has navigation and communication capabilities
  – emphasis of our approach is on arriving at a model-based, optimal sensor repositioning during the search

• Development of a finite-volume computational method on adaptive grids, that provides in real-time the solution (process-state estimate) of the 3D advection-diffusion PDE with variable diffusivities and ambient wind.
  – the unstructured grid allows modeling of complex boundaries (e.g. objects) and will be adapted with local refinement and coarsening during the process-state estimation, in order to improve accuracy and efficiency.
Overview

Mobile Sensing Agents with Wireless Communication

Computational Base Station with wireless communication

Moving Source
Governing Equation

Atmospheric Advection Diffusion Equation

\[
\frac{\partial c}{\partial t} + W_x \frac{\partial c}{\partial X} + W_y \frac{\partial c}{\partial Y} = \frac{\partial}{\partial X} \left( \kappa_{XX} \frac{\partial c}{\partial X} \right) + \frac{\partial}{\partial Y} \left( \kappa_{YY} \frac{\partial c}{\partial Y} \right) + S(t, X, Y)
\]

- \(c(t, X, Y)\) : concentration
- \(W_x, W_y\) : advection (wind)
- \(\kappa_{XX}, \kappa_{YY}\) : eddy diffusivity
- \(S(t, X, Y)\) : source term

Assumptions
- diffusive point source
- eddy diffusion dominates
- no chemical reactions
- minimal diurnal variations for timescales of interest

[Seinfeld 2006, Arya 1999]
applications with similar approach

- Moving Aerial Source
  - Aircraft Exhaust
  - Toxic Releases (crop duster)
  - Single Release (puff)
- Rising plumes
  - Accident (crash, leak, etc)
  - Moving ground source (car)
  - Smoke stacks
### SAV Motion

#### 2D Dynamic Equations

\[ M\dddot{X} = \cos(\psi(t))\tau_l(t) - \dot{\psi}(t)v(t)\sin(\psi(t)) \]

\[ M\dddot{Y} = \sin(\psi(t))\tau_l(t) + \dot{\psi}(t)v(t)\cos(\psi(t)) \]

\[ I\dddot{\psi} = \tau_a(t) \]

#### Velocity Limitations

\[ v_{min} \leq v \leq v_{max} \]

\[ -\omega_{\psi,max} \leq \omega_{\psi} \leq \omega_{\psi,max} \]

#### Input Limitations

\[ \tau_l : \quad \tau_{l,\min} \leq \tau_l^c \leq \tau_{l,\max} \]

\[ \tau_a : \quad -\tau_{a,\min} \leq \tau_a^c \leq \tau_{a,\max} \]

Input Limitations are based on the physical UAVs in the U.S. Air Force (AF.mil).
System equations

\[
\frac{\partial c(t, X, Y)}{\partial t} = A(X, Y)c(t, X, Y) + B\left( X_c(t), Y_c(t) \right)f(t), \quad c(0, X, Y) = c_0(X, Y)
\]

\[
B\left( X_c(t), Y_c(t) \right) = \delta(X - X_c(t))\delta(Y - Y_c(t))
\]

\[
y\left( t; X_s(t), Y_s(t) \right) = c\left( t, X_s(t), Y_s(t) \right) = \int_0^X \int_0^Y \delta(X - X_c(t))\delta(Y - Y_c(t))c(t, X, Y)dXdY
\]

- \( c(t, X, Y) \): concentration at time \( t \) and spatial coordinates \( X \) and \( Y \)
- \( f(t) \): source material release rate (assumed constant)
- \( B(X_c(t), Y_c(t)) \) source term at location \( X_c(t) \) and \( Y_c(t) \)
- mobile sensor has spatial coordinates \( X_s(t) \) and \( Y_s(t) \)
- sensor guidance equivalent to choosing \( X_s(t) \) and \( Y_s(t) \)
System equations

• view “plant” equation as an evolution equation in a Hilbert space

\[
\frac{dx(t)}{dt} = Ax(t) + B(\theta_c(t))f(t); \quad \theta_c(t) = (X_c(t), Y_c(t))
\]

\[
y(t; \theta_s(t)) = c(t, \theta_s(t)) = C(\theta_s(t))x(t); \quad \theta_s(t) = (X_s(t), Y_s(t))
\]

• \(A\) is the (elliptic) operator associated with advection-diffusion process

• \(B(\theta_c(t))\) operator associated with spatial distribution of the moving source

• \(\theta_c(t) = (X_c(t), Y_c(t))\) current source location

• \(C(\theta_s(t))\) output operator associated with spatial distribution of mobile sensor

• \(\theta_s(t) = (X_s(t), Y_s(t))\) current sensor location

• \(y(t; \theta_s(t))\): concentration at current sensor location \(\theta_s(t)\)– i.e. sensor output
State estimator

\[
\frac{d\hat{x}(t)}{dt} = \left( A - \gamma C^* \left( \theta_s(t) \right) C \left( \theta_s(t) \right) \right) \hat{x}(t) + \gamma C^* \left( \theta_s(t) \right) y(t; \theta_s(t))
\]

- \( \gamma C^* \left( \theta_s(t) \right) \) “collocated” filter gain
- \( B(\theta_s(t)) \) operator associated with estimate of moving source location
  - estimate of source position is assigned to current sensor location
- \( e(t) \): state estimation error;
- \( e(t,X,Y) \) difference between actual and estimated concentrations at time \( t \) and spatial coordinates \( X \) and \( Y \)
Guidance Scheme

\[ \varepsilon(t; X_s(t), Y_s(t)) \triangleq e(t, X, Y) \bigg|_{X=X_s(t), Y=Y_s(t)} \]

\[ \varepsilon_X(t; X_s(t), Y_s(t)) \triangleq \frac{\partial}{\partial X} e(t, X, Y) \bigg|_{X=X_s(t), Y=Y_s(t)} \]

\[ \varepsilon_Y(t; X_s(t), Y_s(t)) \triangleq \frac{\partial}{\partial Y} e(t, X, Y) \bigg|_{X=X_s(t), Y=Y_s(t)} \]

• \( \varepsilon(t,X_s(t),Y_s(t)) \): output estimation error at sensor location \((X_s(t), Y_s(t))\)

• \( \varepsilon_X(t,X_s(t),Y_s(t)) \): \(X\)-gradient of output estimation error at sensor location \((X_s(t), Y_s(t))\)

• \( \varepsilon_Y(t,X_s(t),Y_s(t)) \): \(Y\)-gradient of output estimation error at sensor location \((X_s(t), Y_s(t))\)
Guidance scheme

Lyapunov-based Guidance

\[
V = - \langle x, A_{cl} x \rangle + KE + \left( 1 - \cos \left( \psi - \tan^{-1} \left( \frac{\varepsilon \varepsilon_Y}{\varepsilon \varepsilon_X} \right) \right) \right)
\]

\[
\dot{V} = -\left| A_{cl} x \right|^2 + \begin{bmatrix} \varepsilon \varepsilon_X & \varepsilon \varepsilon_Y & 0 \end{bmatrix} S \mathbf{v} + \mathbf{v}^T \beta_2 \tau + \begin{bmatrix} 0 & 0 & \sin \left( \psi - \tan^{-1} \left( \frac{\varepsilon \varepsilon_Y}{\varepsilon \varepsilon_X} \right) \right) \end{bmatrix} S \mathbf{v}
\]

Control Law

\[
S = \begin{bmatrix} \cos \psi & \sin \psi \\ -\sin \psi & \cos \psi \\ 0 & 0 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} \dot{X} \\ \dot{Y} \end{bmatrix}
\]

\[
\tau_l = k_1 \left( -\varepsilon \varepsilon_X \cos \psi - \varepsilon \varepsilon_Y \sin \psi \right) - k_2 v
\]

\[
\tau_a = -k_3 \sin \left( \psi - \tan^{-1} \left( \frac{\varepsilon \varepsilon_Y}{\varepsilon \varepsilon_X} \right) \right) - k_4 \dot{\psi}
\]

where \( k \) values are user-defined gains.

[Demetriou et al CDC2011]
State gradient estimation

- Guidance scheme requires gradient of the state estimation error, and therefore gradient of the process state

- Direct measurement of this value is not possible with available point sensors.

Sensors that are positioned too closely or too far apart may not accurately capture the gradient of the flow.
Length scale estimation

Example length scale for a puff release at a known time

Advection: \[ l_A = vt \]

Diffusion: \[ l_D = \sqrt{2Dt} \]

\( v \): wind velocity
\( D \): diffusivity
\( t \): time

[Arya, 1999]
Length scale estimation

Proposed length scale estimate for an unknown source type with an unknown release time.

\[
l_X = \left| \left( \frac{1}{\hat{x}_G} \frac{\partial \hat{x}}{\partial X} \bigg|_G \right)^{-1} \right|
\]

\[
l_Y = \left| \left( \frac{1}{\hat{x}_G} \frac{\partial \hat{x}}{\partial Y} \bigg|_G \right)^{-1} \right|
\]

- \( l \): length scale
- \( \hat{x} \bigg|_G \): estimated state (concentration) evaluated at nearest grid
- \( X, Y \): spatial position
- \( G \): nearest grid point

Central difference approach used to calculate estimated gradient at grid point.
Sensing agent configurations

- Axial and lateral distances chosen to ensure Cartesian length scales are met
- Boom configuration with sensors affixed to the agent via a boom of variable length.

\[ d : \text{distance} \quad f : \text{fore} \]
\[ A : \text{axial} \quad a : \text{aft} \]
\[ L : \text{lateral} \quad p : \text{port} \]
\[ s : \text{starboard} \]
Sensing agent configurations

- Autonomous support sensor configuration where multiple UAVs are flying in formation.

\[ d : \text{distance} \quad f : \text{fore} \]
\[ A : \text{axial} \quad a : \text{aft} \]
\[ L : \text{lateral} \quad p : \text{port} \]
\[ s : \text{starboard} \]
Computational grid adaptation

• Autonomous, State Dependent Switching
  – Based on estimated location of source
  – Dimension of system is constant
• Grids and State Matrix ‘A’ calculated *a priori*
  – 9 possible choices are stored

9 Available grids with fine region highlighted.
Multigrid switching

- Grid switches to 1 of 9 stretched grids
- Fine grid is stretched to the area of interest
- 25% of domain area is fine grid
- 25% of domain area is coarse grid

Sample Stretched Grid (#3)
Multigrid switching

\[
\dot{x}(t) = \left( A_p - \gamma C_p^T \left( \theta_s(t) \right) C_p \left( \theta_s(t) \right) \right) \hat{x}(t) + \gamma C_p^T \left( \theta_s(t) \right) y_p \left( t; \theta_s(t) \right)
\]

• convert multigrid switching into switching of a hybrid dynamical system

• \( P \): index set; for each \( p \) in \( P \) consider the system \( (A_p, B_p, C_p) \)

• \( A_1 \ldots A_9 \) different matrix representations of elliptic operator \( A \)

• each \( A_i \) corresponds to a different grid discretization

• switching time instants depend on current sensor position; if sensor is in a region of finer grid, then switch to the \( (A, C) \) that correspond to that grid
## Numerical Results

### Stationary Source

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domain Size</td>
<td>4 km x 4 km</td>
</tr>
<tr>
<td>Diffusivity</td>
<td>20 m$^2$/s</td>
</tr>
<tr>
<td>Advection EW</td>
<td>5.0 m/s</td>
</tr>
<tr>
<td>Advection NS</td>
<td>5.0 m/s</td>
</tr>
<tr>
<td>Simulated Time</td>
<td>500 s</td>
</tr>
<tr>
<td>Sensor Velocity</td>
<td>10 $\leq v \leq$ 30 m/s</td>
</tr>
<tr>
<td>Sensor Turning Rate</td>
<td>$-1.5 \leq \omega \leq 1.5$</td>
</tr>
<tr>
<td>Source Velocity</td>
<td>0 m/s</td>
</tr>
<tr>
<td>Simulated Grid</td>
<td>90 x 90, uniform</td>
</tr>
<tr>
<td>Estimator Grid</td>
<td>30 x 30, switched</td>
</tr>
</tbody>
</table>

![Simulation Trajectories - Stationary Source](image)
Numerical Results

Diagonal Source

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
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<tr>
<td>Domain Size</td>
<td>4 km x 4 km</td>
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<tr>
<td>Advection NS</td>
<td>5.0 m/s</td>
</tr>
<tr>
<td>Simulated Time</td>
<td>500 s</td>
</tr>
<tr>
<td>Sensor Velocity</td>
<td>$10 \leq v \leq 30$ m/s</td>
</tr>
<tr>
<td>Sensor Turning Rate</td>
<td>$-1.5 \leq \omega \leq 1.5$</td>
</tr>
<tr>
<td>Source Velocity</td>
<td>15 m/s</td>
</tr>
<tr>
<td>Simulated Grid</td>
<td>90 x 90, uniform</td>
</tr>
<tr>
<td>Estimator Grid</td>
<td>30 x 30, switched</td>
</tr>
</tbody>
</table>

![Simulation Trajectories - Diagonal Source](image)
Numerical Results

Diagonal Source – Actual State and Estimated State
# Numerical Results

## Circular Source

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domain Size</td>
<td>4 km x 4 km</td>
</tr>
<tr>
<td>Diffusivity</td>
<td>20 m$^2$/s</td>
</tr>
<tr>
<td>Advection EW</td>
<td>5.0 m/s</td>
</tr>
<tr>
<td>Advection NS</td>
<td>5.0 m/s</td>
</tr>
<tr>
<td>Simulated Time</td>
<td>700 s</td>
</tr>
<tr>
<td>Sensor Velocity</td>
<td>$10 \leq v \leq 30$ m/s</td>
</tr>
<tr>
<td>Sensor Turning Rate</td>
<td>$-1.5 \leq \omega \leq 1.5$</td>
</tr>
<tr>
<td>Source Velocity</td>
<td>15 m/s</td>
</tr>
<tr>
<td>Simulated Grid</td>
<td>90 x 90, uniform</td>
</tr>
<tr>
<td>Estimator Grid</td>
<td>30 x 30, switched</td>
</tr>
</tbody>
</table>
Numerical Results

Circular Source Grid Adaptation
Numerical Results
Stationary Source with Gradient Estimation

Point Gradient

Estimated / Measured
Numerical Results

Diagonal Source with Gradient Estimation

Point Gradient

Estimated / Measured
Length Scales

Length Scales Used

Stationary Source

Diagonal Source
Summary

• Integrated sensor position (on board SAV) into spatial process at which it collects measurements
  – Sensor location is part of the process-parameterized output operator
• Linked vehicle motion to estimation scheme
  – Sensing aerial vehicle motion directly linked to estimator performance
• Developed a model-based estimation scheme that provides the sensor’s spatial relocation employed to estimate the process state
  – Spatial gradient-based guidance
• Developed a finite-volume adaptive multi-grid computational method that provides in real-time the solution (process-state estimate) of the 2D advection-diffusion PDE with variable diffusivities and ambient wind
  – Implement a grid adaptation with local refinement and coarsening during the process-state estimate (switching of state matrices)
Current/future Work

- **Simulations**
  - Incorporate 3D dynamics of SAV into the 3D model-based estimation scheme simulations with enhanced controller
  - Extend state gradient estimation to 3D case

- **Source Modeling**
  - source dynamic motion
  - different source release rates and types

- **Sensor Modeling**
  - gradient estimation
  - sensor dynamics and sensor motion (Lagrangian sensor)

- **Multiple sensors in network**
  - coordination
  - collision avoidance
  - consensus